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What is Quantum Chaos?

Preface

This issue of the *Journal of Statistical Physics* is devoted to the theme of what is loosely (but colorfully) called Quantum Chaos. The idea for this special issue grew out of a symposium which was held at Rutgers as part of the 65th Statistical Mechanics Meeting in May 1991. We have tried to ensure that the papers presented here have a greater pedagogical content than usual and also that they provide sufficient background material.

Before discussing the question raised in the title, which we should warn the reader is only partially answered here, we have to say something about classical or deterministic chaos. Though building on a foundation a century old, work in dynamical systems in the last decades has resulted in a shift in perspective in physics and in science as a whole. The existence of chaos in classical mechanics has challenged many of our long-held beliefs about "predictability" in deterministic systems. The time evolution of even very simple systems having only a few degrees of freedom typically exhibits complicated "unpredictable" behavior. This ubiquitousness surprised us even more because of how well all the "usual" problems of classical mechanics had seemed to fit the old paradigm. Clearly this was due to our choice of problems. We only tackled those which we could solve, and in the absence of computers those were the non-chaotic ones. Still, given the work of Poincaré, Hill, Hadamard, etc., and our everyday experiences of chaos, how could we physicists have mostly missed it?

In any case, the shift in perspective opened the way for a more penetrating study of truly complex systems, turbulent fluid flow being one example. These problems were long avoided by many physicists because they were deemed messy extensions of known microscopic laws. However, within the "chaos view" of nature, complex and unexpected behavior is "emergent" from simple elements or principles, and an inspection of the microscopic laws does not readily reveal or shed light on this behavior. Within this complexity, however, there lies a deeper order, e.g., weather and billiards both have similar sensitivity to initial conditions. One can even be more quantitative and find universality classes which relate chaotic phenomena occurring in very different situations. All this of course looks

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familiar to the statistical physicist, and so our statistical mechanical view of the universe remains central to our understanding of nature.

But where does this chaos in classical systems come from? Clearly it arises from the nonlinearity in the equations of motion. Attempts to linearize everything led to the integrable or "harmonic oscillator" view of typical behavior. On the other hand, we all know that we live in a world governed by quantum mechanics. Here it is the wave function which is generally thought to contain the complete description of the system, replacing the phase space point of classical theory. The evolution of the wave function as given by the Schrödinger equation is linear. (In fact, any nonlinear correction to quantum mechanics would have to be exceptionally small to be consistent with recent experiments.) Thus quantum mechanics should not contain any chaos. The lack of chaos in the Schrödinger evolution can be made quite precise: the time evolution of the wave function or density matrix of an isolated, spatially bounded, quantum system is almost-periodic and is not sensitive to initial conditions.

Unlike the exponentially divergent trajectories of classically chaotic systems, quantum dynamics is more stable to small perturbation and is marked by pronounced recurrences as dramatically exhibited in the so-called collapses and revivals of the wave function. This linear evolution is completely consistent with chaotic behavior of suitable quantum observables for "short" times and these times can become *very large* indeed for macroscopic systems. Chaos can also occur when part of a quantum system can be regarded as classical, leading to a Schrödinger equation with a time-dependent external field. While this certainly puts to rest the concerns or "problems" raised by some people about the compatibility between quantum mechanics and chaos, it does not give a direct quantum explanation of the origin of chaos—comparable to that of the nonlinearity of the classical equations of motion—without involving a classical limit.

An important question then is whether there is "true" chaos in quantum mechanics or whether some "classical limit" is in fact necessary for the emergence of chaos in a quantum world? While the second possibility presents no significant problem (other than understanding of *how* the classical behavior emerges in a quantum world) for those manifestations of chaos of which we have direct experience, e.g., turbulence or the motion of a tossed coin, it probably does not tell the whole story. It would in fact be quite surprising if it did, since on the microscopic level the observational predictions of quantum mechanics are generally of a stochastic nature and thus presumably more chaotic than those of classical mechanics. Yet classical mechanics is where the chaos seems to be.

The origin of this apparent paradox possibly lies in the fact that we have been dealing exclusively with the evolution of an isolated system. Now the construction of a subsystem which one can treat as "effectively isolated" is much more problematic in quantum mechanics than in classical theory—a fact which manifests itself in the dichotomy between the Schrödinger evolution and the (random) reduction of the wave function which occurs in "observations." This wave function entanglement aspect of quantum chaos, indeed of all behavior, appears to us to be tied up with very fundamental aspects of quantum theory and still needs a lot of investigation—it is barely touched upon in this volume.

Indeed, even the question raised earlier, of "How does classical chaos emerge from quantum mechanics?"—a question about the correspondence principle—is often overlooked. Instead, the principal question explored here and in current research in quantum chaos in general is "What are the quantum manifestations of classical chaos?" Answering this question leads to valuable insights into the properties of the stationary states and dynamics of a variety of systems, including, for example, highly excited molecules. These insights generally come from exploring the quantum manifestations of classical chaos through a semiclassical viewpoint, in which objects have many of their classical attributes, but also carry phase information. It is surprising that this is possible, since the chaotic classical dynamics would seem to prevent application of methods like the WKB approximation. However, there are alternative quantization procedures, such as the Gutzwiller trace formula, that do the trick. These provide information on quantities such as the density of states, localization of the eigenfunctions ("scarring"), and spectra in terms of periodic or spatially self intersecting orbits. It has surprised many that the description obtained in this way is quite accurate in predicting the true quantum behavior, even quite far from the classical limit.

To make better contact with classical chaos, it is important to have systems that can be fully tackled quantum mechanically as well. Such systems, not surprisingly, have been simple, model systems with few degrees of freedom. These are, in fact, often the same ones as are treated semiclassically. One example, the microscopic analogy of the dynamics of an asteroid subject to periodic perturbation by Jupiter (a chaotic system), is the hydrogen atom in a strong radio frequency field. The question then is: If the classical dynamics of the Bohr atom exhibits chaotic behavior, what happens to real atoms? Because examples like this are simple and clearly quantum mechanical, atomic physics is a natural place to look for quantum chaos. However, other areas, such as nuclear physics, and other models, such as kicked rotors, all play important roles in providing simple examples.

These models tell us that the statistical behavior of the quantum energy levels changes as the classical system passes from regular to chaotic.

Level repulsion is found, leading to regions of complicated avoided crossings. In this way chaos appears to allow global energy flow through state space by the coupling of many states of differing symmetry of the "unperturbed" Hamiltonian. The corresponding macroscopic view of such energy flow is via the phase space structure: stable periodic orbits and nearby quasiperiodic orbits are dynamical barriers. As these orbits break up, global energy flow becomes possible. That tunneling through these dynamical barriers is fundamentally different from ordinary potential tunneling is among the topics addressed in this volume. Energy level statistics thus provide a quantum signature and the beginnings of a microscopic explanation for the emergent classical behavior: Wigner–Dyson statistics (includes GOE, GUE, and GSE) indicate chaos in the classical system. Poisson statistics (with its associated level crossings) indicates a regular classical system. There are many other quantum manifestations of classical chaos that have been studied. This volume is rich in examples. One of the more intriguing suggestions is that chaos plays a role in irreversibility and dissipation.

The program of the symposium which inspired this issue is included here so that the reader might get the flavor of the many topics addressed. We hope that the papers presented here further spark the readers' curiosity and delight in the subtleness of nature and inspire her or him to address the deeper question of "Under what circumstances is there chaos (and in what sense) in 'pure' quantum mechanics?"

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